Phasing Problem-Based Teaching into a Traditional Educational Environment

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You may want to try to teach mathematics through problem solving but are working in a traditional teaching environment that seems incompatible with these ideas. We believe that these realities can indeed coexist, although you may need to phase in the new practices gradually. What determines the pace with which you try out new ideas? Perhaps the easiest answer is "Make changes as you are comfortable with them," but the most comfortable stance could be not to change at all. So we suggest an alternative: "Make changes when you are only slightly nervous about them." The extent of your nervousness will be determined not only by how well you think you are teaching but also by the anxiety levels of your students, colleagues, administrators, and students' parents.

Here we address five of the many dimensions of teaching you will be considering: (1) articulating the mathematics, (2) structuring class sessions, (3) sources of problems, (4) working in groups, and (5) changes in the teacher's role. These dimensions of teaching through problem solving are intertwined; success in phasing in one dimension will allow you to progress more easily in other areas.

Articulating the Mathematics

What mathematics are you trying to teach in a lesson? An easy answer, such as "linear equations," is too vague. We find that being more specific helps us teach better, for example, "Linear equations have straight-line graphs," or "Linear equations model constant-rate growth," or "The balancing method is one way of solving a linear equation."

How might you phase in a more specific articulation of the understandings you want to develop? Begin by studying your current textbook. Study other texts, too, including materials from college or in-service courses, to understand the ideas more deeply by seeing them from different viewpoints. Study more mathematics to see the ideas in a broader context, even if you will not be teaching it. (E.g., Does knowing that a linear transformation defined on a vector space preserves lines deepen your understanding of linear equations?) Your articulation can improve through the rest of your career. Later in life, your "better" summary of the foregoing outcome might become "Arrow diagrams and working backward can be used to solve equations that arise in predicting the result of linear growth."

As you become more comfortable articulating the important ideas you want students to understand, you can begin to consider not only content goals but also what you are teaching about mathematical processes. For example, you might articulate the goal "Mathematical modeling is useful but not an entirely algorithmic process" for the same lesson as the content goal above.

As another example, the topic of the day in a traditional curriculum might be the formula for finding the area of a parallelogram. You could write your goal as "The area of a parallelogram is *LW*." Better, however, would be "The area of a parallelogram can be found by multiplying the length of one side by the length of an altitude to that side." Even better might be "One of several ways of finding the area of a parallelogram is to multiply the length of any side by the length of an altitude to that side." To teach to this last goal, you will have to plan for students to see more than one way to determine the area. For example, you could ask them, without any introduction, to find the area of the shape in figure 13.1 in as many ways as they can. This approach can accomplish several process goals, such as "More than one way can be used to solve at least some mathematics problems," "Different degrees of precision in recording data can lead to different answers," and "Looking at a problem from a different point of view can help solve it!"

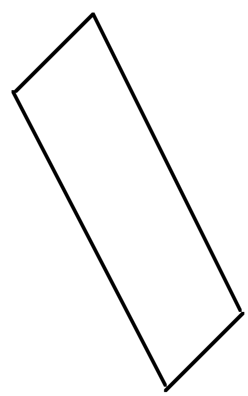

Fig. 13.1

Articulating the mathematics is important whether you are teaching in a traditional or a problem-based way. Articulating content goals that include specific process goals, however, leads naturally to the question of how to accomplish those goals. How can you move your traditional classroom toward a problem-based environment in which students might learn more about both mathematical content and mathematical processes?

Structuring Class Sessions

In many U.S. mathematics classrooms, we first tell students how to solve a particular type of problem, next show them some examples, and then have them practice on more problems of that type. Many content and process goals, however, can be taught more effectively with a different structure: one in which we first give students a problem that they do not know how to solve, then let the mathematical ideas arise as needed to solve that problem. As Stigler and Hiebert (1999) report, class sessions in Japan are often structured this way.

So how might you phase in this change in how your class sessions are structured? At first, you might just pose a problem that you will later use as an example, then go on with your lesson. Gradually let students have more time to think about the problem and to discuss it with a neighbor or a small group. Eventually, you hope to arrive at something like the SPOSA model of problem-based teaching:

> ***S***et the context. Remind students in a sentence or two of what they have been doing in recent class sessions, and if needed, set the stage for the new problem.
>
> ***P***ose the problem, defining any terms that might not be understood by some students. (Be especially sensitive to students who do not understand English well.) Start the students working in their groups.
>
> ***O***bserve and listen to students while they work. Decide which students you will ask to present what, and make some tentative notes on the points that you want to arise during their presentations or in your concluding comments. Observe students' presentations, creating as much safety as you can for students to critique one another's ideas and ask questions.
>
> ***S***ummarize, with the help of the class, what has been done and the mathematical points that have arisen, using the mathematical goals you articulated. Introduce vocabulary for the new concepts.
>
> ***A***ssess the learning and the lesson. For the remainder of the class period, have students assess their learning through writing about it or beginning work on follow-up problems. These problems should not just reiterate the ideas being taught, but rather, should build on them. Work on extensions of the problems can serve as homework. Meanwhile, you can make quick notes about what went well and what changes you will make the next time you teach the problem.

For example, suppose you are introducing vertex-edge graphs. In planning, you search for a problem that will engage students. You look several sections ahead in your own textbook, through other textbooks, and on the Internet. You get ideas and reject them. Finally you settle on an idea and work hard to phrase it carefully, hoping that it will be successful enough to use again. And you ponder all the ways you can think of to approach the problem.

> ***S*** This problem is the introduction to this topic, so students have done no earlier work on the topic to connect with. However, you might say a little about painting lockers and the importance of efficient planning to pique students' interest in the problem context.
>
> ***P*** Here is your problem, adapted from Coxford et al. (1999, pp. 250 ff).
>
> You have a summer job painting lockers in a school building. On one floor, the lockers (shown by the thick lines in fig. 13.2) are located in the hallways along the walls around eight classrooms and along an outside wall.

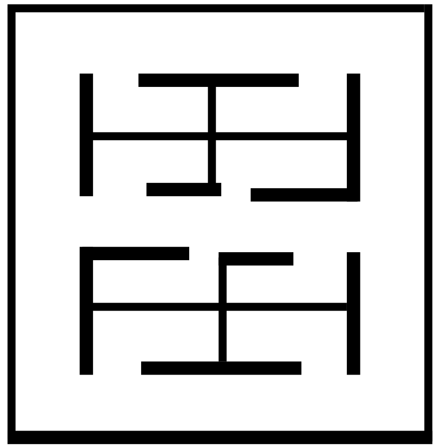

Fig. 13.2

> Because you are moving bulky equipment, the lockers in the center hall must be painted one row at a time.
>
> In what order should you paint the lockers to avoid moving the equipment any farther than necessary?

After students have read the problem, you might make sure they know what the word *bulky* means and understand why the equipment should not be dragged back and forth across a hall.

O As groups work, observe how they represent the problem in various ways. When they present their ideas to the full class, observe how well both the presenters and the audience members exhibit understanding of previously encountered ideas.

S Introduce the terminology *graph*, *vertex*, *edge*, *degree*, *Euler circuit*, and so on, to help with the discussion. With your students' help, articulate the ideas you are trying to teach: "Movement from one place or task to another can often be modeled by vertex-edge graphs," "A graph has an Euler circuit if and only if all its vertices have even degree," "A good problem-solving technique is to look for patterns in simpler problems," and so on. Pose extension problems, such as, "But *why* are the degrees of vertices related to Euler circuits?" or "What if you wanted to touch every vertex exactly once instead of every edge exactly once?"

A You might give out note cards to the students and ask them to write a sentence or two about what they learned and about what remains confusing to them. Meanwhile, make your own notes of what you would do differently next time and how well various students demonstrated understanding of the mathematical ideas, facility at problem solving, and skills in doing group work.

Even though at first you may designate particular class periods as "SPOSA days," you can let the distinction fade as you begin using the SPOSA techniques more frequently. To aid in teaching both the content and process goals of the mathematics you have articulated, such problem-based structures as SPOSA begin by posing a problem. Where do such problems come from if you are using a traditional curriculum?

Choosing Problems

You can start by going to your textbook. Look over the examples and the exercises. Or find a problem later in the textbook that could promote thinking about several topics. If your curriculum comes with auxiliary materials, consult them for performance assessments or extension problems. Some published standards—from both NCTM (2000) and individual states—have good examples. Talk with colleagues in your school, your district, or your state professional organization; some of them may have collections of good problems. Occasionally you will find a good problem in a puzzle book or problem collection, perhaps on the World Wide Web. A growing database of rich problems is found at Copes (2003).

You may be tempted to use a problem just because it is fascinating, but the most important selection criterion is that the problem will carry the mathematics you want to teach. When you find a fascinating problem that you think will engage your students, study it until you can articulate what mathematics, both content and processes, it involves, and then file it away for when you are teaching those ideas.

For example, suppose that you want to teach these content goals with understanding.

- In a linear growth function, the amount of change is the same in each equal interval of the domain.
- In an exponential growth function, the amount of change in an interval of the domain is a constant multiple of the amount of change in the previous interval of equal length.
- Exponential growth is more dramatic overall than linear growth, even if the initial changes are smaller.

A problem that might serve well for teaching these goals is the two-job problem:

> You have been offered two summer jobs. One begins at $5.50 per hour and increases by 10 cents per hour each week that you remain on the job. The other also begins at $5.50 per hour and increases by 1 cent per hour the first week, 2 cents per hour the second week, 4 cents per hour the third week, 8 cents per hour the fourth week, and so on, the amount of your raise doubling each week. Which job will pay you more for the summer?

In a problem-based classroom, this problem can also help in teaching some process goals, such as these:

- Several ways can be used to solve a mathematics problem.
- Real-life problems are often ambiguous, and solving them requires making assumptions.

What if you want to teach a particular idea and have no relevant rich problems in your storehouse? For example, you may be working with a curriculum that emphasizes basic skills, just as many standardized tests do. You can nonetheless phase in rich problems by enriching some of the problems that are part of the curriculum. For some examples, see Goldenberg and Walter (this volume) and Becker and Shimada (1997). One of the easiest and most effective ways to do so is to ask students to solve a problem in as many ways as they can. Such a challenge can lead to a deepening of their mathematical understanding as students see different perspectives on the same idea and make connections with other ideas. It can also help students who otherwise "freeze up" when they cannot remember "the right way to solve this problem."

For example, a paraprofessional, working with two children who were bored with a drill-and-practice worksheet, simply wrote at the top "Work these problems in as many ways as you can." One of the previously disenchanted students stayed with the worksheet for about 20 minutes. The other was engaged with the problems for an hour and a half, working each problem in several ways, not necessarily including the way the teacher had taught. Even the leanest of problems can be enriched to become more engaging and to foster the deeper understanding that comes through multiple perspectives.

Besides selecting good problems, an essential component of problem-based teaching is observing and listening. You can make observations if the class is working in a traditional way, but usually you can observe and hear more, and the students can learn more, if they are working in groups. How do you phase group work into a traditional environment?

Working in Groups

To phase in group work, you might begin by reading and by attending professional development workshops. If you or your students are uncomfortable with having the class work in groups, you might build comfort by having pairs of students in adjacent seats work together on a focused task for only a minute or two at a time. Later, you can group three or four students occasionally, again for a few minutes at a time. As you extend the amount of time that students work in groups, you might assign roles to students until all are in the habit of contributing. You can also articulate—preferably with the help of the class—rules for group work.

Parents, students, and administrators will need reassurance that students' grades will not be lowered by working in groups. We suggest that you continue to assign grades to individual students rather than groups, but as students acquire skills in group work, you might gradually introduce a group-participation part of their grade. Its weight may increase over grading periods from, say, 5 percent to 15 percent. This grade shows that you believe in the value of learning group skills. You can emphasize that the group participation grade is based on notes you make as you circulate around the classroom while students are working in groups.

You can begin group work even though you have had little experience using this format. But you will be happier if you build up your trust in the group process as well. Gradually take more time to help students address difficulties with working together. When students ask you for answers or hints to mathematical questions, try to avoid responding directly by deflecting their questions to their group members.

Problem-based teaching in a traditional setting begins with articulating mathematics and then finding appropriate problems to be used in a SPOSA-like class structure, usually with students working in groups. Your teaching will be most effective, though, if you also alter your role in the classroom. (See the chapters by Grouws and by Rasmussen, Yackel, and King in this volume for other perspectives on the teacher's role.)

The Teacher's Role

In a traditional environment, a teacher acts as a dispenser of knowledge. A more effective role for you might be described as a manager of investigation teams. Imagine that you are employed by a company or governmental agency, and you supervise teams of employees whose work is to solve various problems that come your way. What would your job entail?

Some of your tasks would be similar to what you probably already do, such as assigning problems to teams, evaluating the workers, and dealing with your supervisors. Other managerial aspects that you might find to be less familiar in a traditional educational environment may have to be phased in over time.

- **Good managers avoid micromanaging.** Begin to trust the student groups to organize their work, to think creatively, and to self-correct their ideas. Gradually you can intervene less and less, until you are focusing only on helping the groups work more effectively. When groups make presentations to the class, increasingly withhold your judgment, letting other students critique each presentation before you ask clarifying questions.

- **Even the best managers do not know the outcomes.** In business, money would be wasted if teams worked on problems that managers could already solve. In education, though, you can indeed solve the problems yourself. If you work toward more and more openness, however, you cannot really anticipate all methods and outcomes. As you learn to resist students' manipulations to get you to show them a solution, and as you encourage them to think creatively yet critically, they will begin to produce ideas you have not anticipated—ideas that you can then use to bring about deep understanding.

- **Good managers are experienced investigators**. Although you may be increasingly surprised by the outcomes, you have experience as a problem solver. You can offer suggestions, not to give omniscient "hints" but to provide good thinking strategies. "You know, the problem seems pretty complicated to me. I often find it helpful to work on a simpler problem first. Have you tried that?" Or "Do I remember that you reported on a similar case before?" Or "I believe I heard that the group over there had an idea about that. Why don't you send a representative over to find out if I heard correctly?" "Are you trying to say that this is a method to trisect every angle? Do the rest of you agree?"

- **Good managers hold workers accountable.** You pick individuals or groups to present their ideas to the entire class. Increasingly, part of the grade you give each student reflects your assessment of that student's contributions to group work.

- **Good managers give appropriate praise and encouragement.** You cannot very well praise students for being "on the right track" while you are in the role of not knowing how to solve the problem. You can, however, praise good thinking and cooperation when you see it. You can also encourage students to learn from mistakes and to persevere despite setbacks.

- **Good managers are not the center of attention.** As you observe group work and presentations, begin sitting down or kneeling as much as possible to remove attention from yourself. Consciously try to disappear from students' awareness. Try to suppress the desire to be the focus of attention. Move toward letting students be more independent thinkers and learners.

- **Good managers take a long-term perspective.** Become more familiar with the entire curriculum and where the problem you have just assigned lies in that context. Become aware of the extent to which you must transcend the focus on solving this particular problem and build students' problem-solving and group skills so that they can deal successfully with the mathematics that lies ahead. Think more about the students' life spans and of how important the process skills and habits of mind will be in those lives.

- **Good managers manage.** They do not manage the individuals' thought process, but they manage the groups of workers so they work productively. Begin focusing on doing all that you can to help the groups function well together. Keep control of the working atmosphere so that individual workers feel confident that they can contribute and so that nobody distracts others from working. Think more about also controlling the order of group presentations so that students get ideas from one another.

Synergy

As you change your role and the structure of your class sessions, you loosen your control over students' thinking, so they will be generating ideas that you had not thought of before. As a good manager, you are enthusiastic rather than threatened, and you ask questions until you and the students better understand the proposed ideas. This questioning deepens your own understanding of the mathematics and helps in your articulation of the mathematical content and process goals. In the future, you will understand more deeply the foundational ideas, so that you can choose or generate rich problems that get at those ideas. Over time, you build your students' confidence and engagement so that your role and the structure of the class can be adapted to support learning through problem solving.

Some progress can be made in a term, and a good bit can occur over a year. Your own mathematical understanding, however, continues to deepen as you work with students for many years. And your skill as a manager of investigation teams grows as well, so that you and your students continue to learn in increasingly rich environments.

End Notes

[1]Our management analogy is not meant to deny the important differences between schools and companies, but only to point out that a middle manager's role in a company has interesting commonalties with the role of a teacher who is teaching mathematics through problem solving.

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